

Communication channel of fermionic system in accelerated frame

Jinho Chang and Younghun Kwon*

Department of Physics, Hanyang University, Ansan, Kyunggi-Do, 425-791, South Korea

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In this article, we investigate the communication channel of fermionic system in an accelerated frame. We observe that at the infinite acceleration, the mutual information of single rail quantum channel coincides with that of double rail quantum channel, but those of classical ones reach different values. Furthermore, we find that at the infinite acceleration, the conditional entropy of single(or double) rail quantum channel vanishes, but those of classical ones may have finite values. In addition, we see that even when considering a method beyond the single mode approximation, the dual rail entangled state seems to provide better channel capacity than the single rail entangled state, unlike the bosonic case. Moreover, we find that the single-mode approximation is not sufficient to analyze the communication channel of fermionic system in an accelerated frame.

I. Introduction

The quantum communication locates in the center of research about quantum information theory. The quantum communication is to find a way to communicate a quantum information between partners. The effectiveness of the quantum communication is usually measured by some suitable measures, i.e. mutual information and conditional entropy, etc.

Along the line, the quantum communication in relativistic quantum information is to understand the effectiveness of quantum communication when parties that may share the quantum channels with partner in an inertial frame moves in an accelerated frames[1][2][3][4][5][6]. Actually, in view of the entanglement behavior of relativistic quantum information, there exists the striking effect that even though the entanglement in bosonic system disappears in the infinite acceleration, the entanglement in fermionic system may persist even in the infinite acceleration[3].

Recently the studies for communication channel of bosonic system in an accelerated frame have been understood[10][11], where the single and double rail encodings were introduced. That is, through a bosonic channel, they try to figure out what is the best method to communicate the quantum information between one in an inertial frame and the other in an accelerated frame. Likewise, it is important to understand the communication channel of fermionic system specially when one of the partners is in an accelerated frame. Up to our knowledge, the study for the channel of the fermionic system was performed only by the single mode approximation which seems not to be a perfect one. In fact, a physical ordering beyond single mode approximation[7][8][9] was recently proposed. So using the physical structure, we investigate the quantum(classical) communication channel such as Φ^+ and $\Phi^*(\rho^s, \rho^d)$, when one partner moves in a uniformly accelerated frame. We find out that at the infinite acceleration, the mutual information of single rail quantum channel coincides with that of double rail quantum channel even though those of classical ones reach different values. Furthermore, we observe that at the infinite acceleration, the conditional entropy of both

single and double rail quantum channel vanishes, nevertheless those of classical ones may have finite values. In addition, we can see that even when considering a method beyond the single mode approximation, the dual rail entangled state seems to provide better channel capacity than the single rail entangled state, unlike the bosonic case. Moreover, we note that the single-mode approximation is not sufficient to analyze the communication channel of fermionic system in an accelerated frame.

The organization of this article is as follows. In section II, we will briefly introduce how to describe the fermionic system in a non-inertial frame. In section III we will investigate the channel capacity of fermionic system in an accelerated frame. In section IV we will conclude and discuss our results.

II. Accelerated Frame

The accelerated frame can be described by the Rindler coordinate (τ, ς, y, z) instead of Minkowski coordinate (t, x, y, z) , having a relation with

$$ct = \varsigma \sinh\left(\frac{a\tau}{c}\right), x = \varsigma \cosh\left(\frac{a\tau}{c}\right) \quad (1)$$

Here a denotes the fixed acceleration of the frame and c is the velocity of light. Eq(1) only covers the region (I) in right edge. The region (II)(the left edge) can be covered by $ct = -\varsigma \sinh\left(\frac{a\tau}{c}\right), x = -\varsigma \cosh\left(\frac{a\tau}{c}\right)$.

The field in Minkowski and Rindler spacetime can be written as

$$\begin{aligned} \phi &= N_M \sum_i (a_{i,M} v_{i,M}^+ + b_{i,M}^\dagger v_{i,M}^-) \\ &= N_R \sum_j (a_{j,I} v_{j,I}^+ + b_{j,I}^\dagger v_{j,I}^- + a_{j,II} v_{j,II}^+ + b_{j,II}^\dagger v_{j,II}^-) \end{aligned} \quad (2)$$

Here N_M and N_R denote the normalization constants. Also $v_{i,M}^\pm$ ($v_{i,I}^\pm$ and $v_{i,II}^\pm$) mean(s) the positive and negative energy solutions of the Dirac equation in Minkowski spacetime(Rindler spacetime), which can be obtained with respect to the Killing vector field in Minkowski spacetime(region I and II). And $a_{i,\Delta}^\dagger$ ($a_{i,\Delta}$) and $b_{i,\Delta}^\dagger$ ($b_{i,\Delta}$) are the creation(annihilation) operators for the positive and negative energy solutions(particle and antiparticle), where Δ denotes M, I, II . A combination of Minkowski

*Electronic address: yyhkwon@hanyang.ac.kr

mode, called Unruh mode, can be transformed into monochromatic Rindler mode and shares the same vacuum. It holds the relation

$$A_{\Omega,R/L} \equiv \cos r_{\Omega} a_{\Omega,I/II} - \sin r_{\Omega} b_{\Omega,II/I}^{\dagger}$$

Here $\cos \gamma_{\Omega} = (e^{\frac{-2\pi\Omega c}{a}} + 1)^{-1/2}$. Actually we may obtain more general relation such as

$$a_{\Omega,U}^{\dagger} = q_L(A_{\Omega,L}^{\dagger} \otimes I_R) + q_R(I_L \otimes A_{\Omega,R}^{\dagger}) \quad (3)$$

, which goes beyond the single mode approximation. Using this relation, in case of Grassmann scalar, the Unruh vacuum can be given by

$$\begin{aligned} |0_{\Omega}\rangle_U &= \cos^2 \gamma_{\Omega} |0000\rangle_{\Omega} - \sin \gamma_{\Omega} \cos \gamma_{\Omega} |0011\rangle_{\Omega} \\ &+ \sin \gamma_{\Omega} \cos \gamma_{\Omega} |1100\rangle_{\Omega} - \sin^2 \gamma_{\Omega} |1111\rangle_{\Omega} \end{aligned} \quad (4)$$

Here we use the notation $|pqmn\rangle_{\Omega} \equiv |p_{\Omega}\rangle_I^{\dagger} |q_{\Omega}\rangle_{II} |m_{\Omega}\rangle_I |n_{\Omega}\rangle_{II}^{\dagger}$. The two different one-particle states can be obtained as

$$\begin{aligned} |1_{\Omega}^{+}\rangle_U &= q_R(\cos \gamma_{\Omega} |1000\rangle_{\Omega} - \sin \gamma_{\Omega} |1011\rangle_{\Omega}) \\ &+ q_L(\sin \gamma_{\Omega} |1101\rangle_{\Omega} + \cos \gamma_{\Omega} |0001\rangle_{\Omega}) \\ |1_{\Omega}^{-}\rangle_U &= q_L(\cos \gamma_{\Omega} |0100\rangle_{\Omega} - \sin \gamma_{\Omega} |0111\rangle_{\Omega}) \\ &+ q_R(\sin \gamma_{\Omega} |1110\rangle_{\Omega} + \cos \gamma_{\Omega} |0010\rangle_{\Omega}) \end{aligned} \quad (5)$$

Here we consider q_R and q_L as real number. From now on, for simplicity, the index Ω will be omitted. It should be noted that the physical ordering of the fermionic system was recently introduced by [8][11][9]. So in this report we will use the physical structure proposed by [7][8][9].

III. Classical and quantum channel for fermionic system in accelerated frame

A. The 2 party generalized entangled state Φ^{+}

First of all we consider the generalized entangled states Φ^s state and Φ^d for the ingredient of quantum channel.

$$|\Phi^s\rangle = \cos \alpha |0\rangle_M |0\rangle_U + \sin \alpha |1\rangle_M |1^{+}\rangle_U \quad (6)$$

$$|\Phi^d\rangle = \cos \alpha |1^{+}\rangle_M |1^{+}\rangle_U + \sin \alpha |1^{-}\rangle_M |1^{-}\rangle_U \quad (7)$$

First of all, it should be noted that eq(7) can be considered as a fermionic analog of dual rail encoding for bosonic system. Here two parties Alice and Bob prepare generalized entangled states such as Φ^s and Φ^d in inertial frames and afterward Bob moves in a uniformly accelerated frame. Here we assume that Bob and anti-Bob's detector cannot distinguish between his particle and antiparticle. Since Bob has inaccessible part due to his acceleration, when we go beyond the single mode approximation, the states that Alice and Bob(in Bob's region I) may share can be found by tracing out the inaccessible part(Bob's region II), which we denote as $\rho_{AB_I}^{\Phi^s}$ and $\rho_{AB_I}^{\Phi^d}$. In the same way since Bob has inaccessible part due to his acceleration, when we go beyond the single mode approximation, the state that Alice and antiBob(in Bob's region II) may share can be obtained by tracing out the

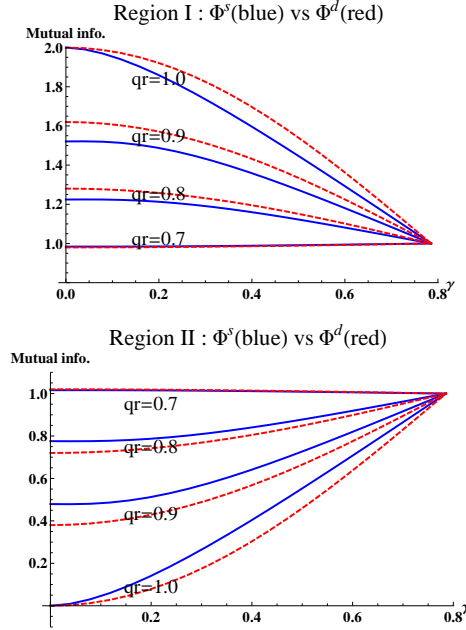


FIG. 1: The mutual information of $\rho_{AB_I}^{\Phi^s}, \rho_{AB_{II}}^{\Phi^s}, \rho_{AB_I}^{\Phi^d}$ and $\rho_{AB_{II}}^{\Phi^d}$. Part (a)((b)) shows the case of $\rho_{AB_I}^{\Phi^s}$ and $\rho_{AB_I}^{\Phi^d}$ ($\rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$). The solid(dotted) lines denote the mutual information of $\rho_{AB_I}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^s}$ ($\rho_{AB_I}^{\Phi^d}$ and $\rho_{AB_{II}}^{\Phi^d}$). In part (a) the lines from top to bottom denote the mutual information of $q_R = 1, q_R = 0.9, q_R = 0.8$ and $q_R = \frac{1}{\sqrt{2}}$ respectively, when $\alpha = \frac{\pi}{4}$. In part (b) the lines from bottom to top denote the mutual information of $q_R = 1, q_R = 0.9, q_R = 0.8$ and $q_R = \frac{1}{\sqrt{2}}$ respectively, when $\alpha = \frac{\pi}{4}$. $\gamma = \frac{\pi}{4}$ means the infinite acceleration. As it can be seen, the mutual information of $\rho_{AB_I}^{\Phi^s}$ and $\rho_{AB_I}^{\Phi^d}$ ($\rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$) coincides at infinite acceleration.

Bob's region I. Here we denote the state that Alice and antiBob(in Bob's region II) share as $\rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$.

We can consider the classical channels such as

$$\rho^s = \cos \alpha |00\rangle\langle 00| + \sin \alpha |11^{+}\rangle\langle 11^{+}| \quad (8)$$

$$\rho^d = \cos \alpha |1^{+}1^{+}\rangle\langle 1^{+}1^{+}| + \sin \alpha |1^{-}1^{-}\rangle\langle 1^{-}1^{-}| \quad (9)$$

Here ρ_d can be interpreted as fermionic version for classical channel in dual rail encoding. Two parties Alice and Bob prepare the classical channels such as ρ^s and ρ^d in inertial frames and afterward, Bob moves in a uniformly accelerated frame. The classical channels between Alice and Bob(Bob's region I)(or between Alice and anti-Bob(Bob's region II)) can be obtained by tracing out the part of Bob's region II(that of Bob's region I). That is, since Bob has inaccessible part due to his acceleration, when we go beyond the single mode approximation, the classical channels that Alice and Bob(in Bob's region I) may share are $\rho_{AB_I}^s$ and $\rho_{AB_I}^d$. Likewise, the classical channels that Alice and antiBob(in Bob's region II) may share become $\rho_{AB_{II}}^s$ and $\rho_{AB_{II}}^d$. The channel capacity between two party A, B can be found from the mutual information

$$S(\rho_A : \rho_B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \quad (10)$$

Here $S(\rho)$ denotes the Von Neumann entropy given by

$-tr(\rho \log \rho)$. The channel capacity can be obtained by maximum value for the mutual information. Actually, the channel capacity will be useful for evaluating the classical channel. The effectiveness of the quantum channel can be measured by the conditional entropy given by

$$S(\rho_A|\rho_B) = S(\rho_{AB}) - S(\rho_B) \quad (11)$$

The mutual information of $\rho_{AB_I}^{\Phi^s}$, $\rho_{AB_{II}}^{\Phi^s}$, $\rho_{AB_I}^{\Phi^d}$ and $\rho_{AB_{II}}^{\Phi^d}$ can be found in Fig 1. The behavior of the mutual information for those quantum states is studied in terms of γ and q_R . At first, the mutual information for Alice and Bob when $q_R = 1$ or $q_R = 0.9$ or $q_R = 0.8$, decreases as the acceleration increases. And the degree for decrease is different. Furthermore, it is worthwhile to note that the mutual information for $\rho_{AB_I}^{\Phi^d}$ is greater than that of $\rho_{AB_I}^{\Phi^s}$ as the acceleration increases. Actually, in an inertial frame there are four different maximally entangled states, but they are locally equivalent in terms of local unitary transformations. However, in an accelerated frame we may have different entangled states such as Φ^s and Φ^d , which shows different mutual information. Also it should be addressed that the mutual information of each state has the same value at the infinite acceleration regardless of q_R . Part (b) in Fig.1. depicts the mutual information of $\rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$. In part (b) the mutual information for Alice and antiBob when $q_R = 1$ or $q_R = 0.9$ or $q_R = 0.8$, increases as the acceleration increases.

The mutual information for classical channel can be found in Fig 2. As we saw in part (a) in Fig.2, the mutual information of both classical channels for single and dual rail one between Alice and Bob when $q_R = 1$ or $q_R = 0.9$ may decrease as the acceleration increases. And it should be noted that the behavior of mutual information for ρ^s is different from that of ρ^d . Also smaller q_R is lesser the mutual information of both classical channels is. At each q_R the mutual information of ρ^d is larger than that of ρ^s . Part (b) in Fig.2 denote the mutual information of both classical channels for single and dual rail one between Alice and antiBob.

The conditional entropy of $\rho_{AB_I}^s, \rho_{AB_I}^d, \rho_{AB_{II}}^s, \rho_{AB_{II}}^d, \rho_{AB_I}^{\Phi^s}, \rho_{AB_I}^{\Phi^d}, \rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$ can be found in Fig.3. Part (a),(b),(c), and (d) in Fig.3 denote the conditional entropy of $\rho_{AB_I}^s$ and $\rho_{AB_I}^d, \rho_{AB_{II}}^s$ and $\rho_{AB_{II}}^d, \rho_{AB_I}^{\Phi^s}$ and $\rho_{AB_I}^{\Phi^d}$, and $\rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$ respectively. As it can be seen in part (a) and (b) in Fig.3, the conditional entropy of $\rho_{AB_I}^s$ and $\rho_{AB_I}^d$ always possesses positive value. However, part (c) in Fig.3 clearly shows the negative conditional entropy of $\rho_{AB_I}^{\Phi^s}$ and $\rho_{AB_I}^{\Phi^d}$ when the acceleration has finite value. In fact, the conditional entropy can be nicely understood in terms of the strong additivity. The strong additivity can be found from eq(12) such as

$$A(\rho_A, \rho_B, \rho_C) = S(\rho_{AB}) - S(\rho_A) + S(\rho_{AC}) - S(\rho_C) \geq 0 \quad (12)$$

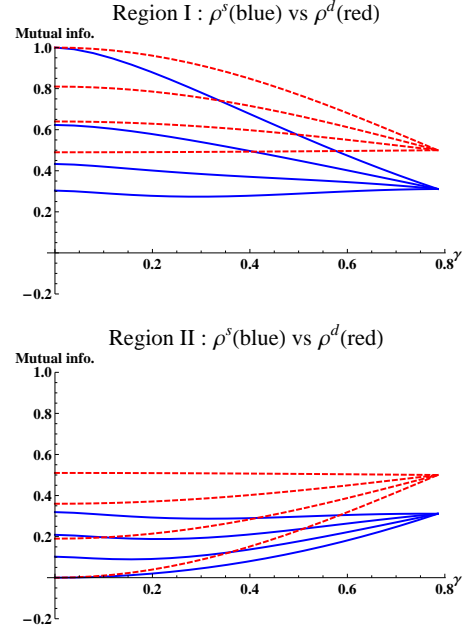


FIG. 2: The mutual information of classical channels $\rho_{AB_I}^s, \rho_{AB_I}^d, \rho_{AB_{II}}^s$ and $\rho_{AB_{II}}^d$. Part (a)((b)) shows the case of $\rho_{AB_I}^s$ and $\rho_{AB_I}^d$ ($\rho_{AB_{II}}^s$ and $\rho_{AB_{II}}^d$). The solid(dotted) lines denote the mutual information of $\rho_{AB_I}^s$ and $\rho_{AB_I}^d$ ($\rho_{AB_{II}}^s$ and $\rho_{AB_{II}}^d$). In part (a) the lines from top to bottom denote the mutual information of $q_R = 1, q_R = 0.9, q_R = 0.8$ and $q_R = \frac{1}{\sqrt{2}}$ respectively, when $\alpha = \frac{\pi}{4}$. In part (b) the lines from bottom to top denote the mutual information of $q_R = 1, q_R = 0.9, q_R = 0.8$ and $q_R = \frac{1}{\sqrt{2}}$ respectively, when $\alpha = \frac{\pi}{4}$. $\gamma = \frac{\pi}{4}$ means the infinite acceleration.

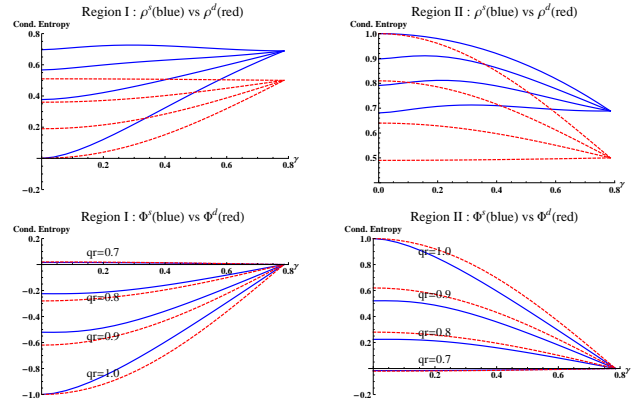


FIG. 3: The conditional entropy of $\rho_{AB_I}^s, \rho_{AB_I}^d, \rho_{AB_{II}}^s, \rho_{AB_{II}}^d, \rho_{AB_I}^{\Phi^s}, \rho_{AB_I}^{\Phi^d}, \rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$. Part (a),(b),(c), and (d) denote the conditional entropy of $\rho_{AB_I}^s$ and $\rho_{AB_I}^d, \rho_{AB_{II}}^s$ and $\rho_{AB_{II}}^d, \rho_{AB_I}^{\Phi^s}$ and $\rho_{AB_I}^{\Phi^d}$, and $\rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$ respectively. $\gamma = \frac{\pi}{4}$ means the infinite acceleration. As it can be seen, the conditional entropy of $\rho_{AB_I}^{\Phi^s}$ and $\rho_{AB_I}^{\Phi^d}$ ($\rho_{AB_{II}}^{\Phi^s}$ and $\rho_{AB_{II}}^{\Phi^d}$) coincides at infinite acceleration.

In order to investigate the strong additivity we consider the Werner state, where a white noise is added to a maximally entangled states. The mixedness of Werner states is parameterized by a single parameter. So we

suppose that two parties Alice and Bob prepare Werner states in inertial frames, and then Bob moves in the uniformly accelerated frame. That is, the initial state of Alice and Bob can be expressed as follows,

$$\rho_W = F|\Phi^s(\alpha = \pi/4)\rangle\langle\Phi^s(\alpha = \pi/4)| + \frac{1-F}{4}\mathbb{I}, \quad (13)$$

where the maximally entangled state is taken from Eq.8 when $\alpha = \pi/4$. Suppose that Bob moves in an accelerated frame. Beyond the single-mode approximation, the state that Alice and Bob(antiBob) share in Bob's region $I(II)$ is obtained by tracing the region $II(I)$. And we denote the quantum state as $\rho_{AB_I}^W(\rho_{AB_{II}}^W)$.

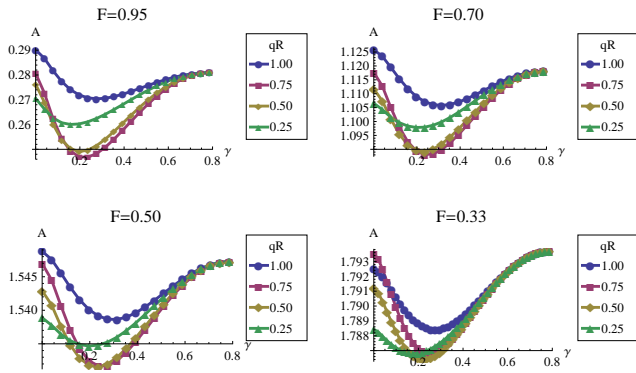


FIG. 4: The strong additivity of $\rho_{AB_I}^W$. Part (a),(b),(c) and (d) of Fig. 3 depict the inequality for conditional entropy of Werner state with $F = 0.95, F = 0.70, F = 0.50, F = 0.33$ respectively. The circle, rectangle, diamond, and triangle denote the strong additivity of $\rho_{AB_I}^W$ at $q_R = 1, q_R = 0.75, q_R = 0.5$ and $q_R = 0.25$ respectively. $\gamma = \frac{\pi}{4}$ means the infinite acceleration.

The strong additivity for Werner state can be seen in Fig.4. It explains the behavior of strong additivity of the state $\rho_{AB_I}^W$. Part (a),(b),(c) and (d) of Fig. 4

depict the inequality for conditional entropy of Werner state with $F = 0.95, F = 0.70, F = 0.50, F = 0.33$ respectively. Specially Part (d) shows why we need a method beyond single mode approximation, where the value at $q_R = 1$ is lesser than that of $q_R = 0.75$ in some region of acceleration.

IV. Discussion and Conclusion

In this article, we investigated the communication channel of fermionic system in an accelerated frame. We considered classical and quantum channels in terms of the single and dual rail encoding. We found that at the infinite acceleration, the mutual information of single rail quantum channel coincides with that of double rail quantum channel, but those of classical ones reach different values. In view of conditional entropy, we observed, that at the infinite acceleration, the conditional entropy of single(or double) rail quantum channel vanishes, even though those of classical ones may have finite values. Furthermore, we saw that even when considering a method beyond the single mode approximation, the dual rail entangled state seems to provide better channel capacity than another maximally entangled state, unlike the bosonic case recently studied by Montero and Martín-Martínez. Moreover, we found that the single-mode approximation is not sufficient to analyze the communication channel of fermionic system in an accelerated frame.

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